JOURNAL OF AIRCRAFT
Vol. 40, No. 3, May–June 2003

Engineering Notes

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Numerical Biobjective Optimal Design of Airfoil and Wing

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Introduction and Background

A MODERN fighter is required to have high aerodynamic performance, that is, high lift with low drag, high aerodynamic efficiency (C_L/C_D) at subsonic speed, and in the meantime low cruise drag at supersonic speed, etc. Stealthy performance has become one of the basic requirements to a modern fighter. The task of a designer today is to shape the aircraft with not only maximum aerodynamic efficiency but also low observability. Up to now, reducing radar cross section (RCS) is the most important part of a low observable technique for a flight vehicle. These requirements derive the development of multiobjective (MO)/multidisciplinary (MD) optimization.

In multiobjective optimization design problems the aim is to minimize (or maximize) a vector function, whose components are individual objectives. There is seldom just one optimal solution, but a family of optimal solutions to this kind of problems. This family is considered as a pareto-optimal set of solutions, which is desirable for many practical engineering designs, because the tradeoff among various objectives can be observed in it. Using traditional optimization approaches, several optimization processes have to be conducted to find the set of pareto-optimal designs. In some cases balanced or nearly equal gains of individual objects are desired, for example, balanced gains of lift and drag are desired for a high aerodynamic performance airfoil or wing. Thus, the goal of MO/MD optimization in these cases is to obtain the required one of their pareto solutions at a minimal computing expense. A computational design of a high-performance airfoil and wing using biobjective (BO)/bidisciplinary (BD) optimization technique is discussed in the present Note.

Optimization Approach

There are numerous available monoobjective methods: deterministic optimization methods (DOM) and genetic algorithms (GA). Monoobjective optimization is a scalar optimization, whereas BO/BD optimization is a vector optimization. To apply the numerous available monoobjective optimization algorithms to BO/BD optimization, the vector optimization problem has to be formulated as a scalar one.

The key of this formulation is to construct a resultant objective function from the individual objectives (in deterministic method) or a suitable fitness function (in genetic algorithm).

A commonly used and well-known approach, named the linear combination of weighted objects optimization (LWO) method, to formulate a vector optimization as a scalar one is taking the weighted sum of individual objective functions, that is, $\sum_{i=1}^{n} C_i obj_i$ as the objective function of MO/MD optimization.[1] Here C_i is called the weighted coefficient, $0 \le C_i \le 1$ and $\sum_{i=1}^{n} C_i = 1$. So the resultant objective function using the LWO method in DOM can be expressed as

$$F = \sum_{i=1}^{n} C_i obj_i \tag{1}$$

The fitness function using LWO method in GA can be expressed as

$$FF = \exp\left(\sum_{i=1}^{n} C_i obj_i\right) \tag{2}$$

The disadvantage of LWO is that its solution is very sensitive to the combination of weighted coefficient, which is mainly determined with user's experience. To improve it, a new method [objective function combination method (OFCM)] for constructing the resultant objective function in DOM or the fitness function in GA is suggested. The general guideline of constructing this kind of function is as follows:

1) The gains of all individual objects can be obtained as equal as possible.

2) The total gain of all objects is expected as large as possible.

In the BO/BD case, according to the guideline, a new constructed resultant objective function can be expressed as

$$F = R^{\beta}\left[(1-\alpha_1)(1-\phi^2)+\alpha_1\right], \qquad 0 \le \phi \le 1, \quad \alpha_1, \beta = \text{const} \tag{3}$$

$$\phi = \begin{cases} \dfrac{4}{\pi}\left|\tan^{-1}\left(\dfrac{GO_2}{GO_1}\right) - \dfrac{\pi}{4}\right|, & GO_1 > 0 \\ 4 - \dfrac{4}{\pi}\left|\tan^{-1}\left(\dfrac{GO_2}{GO_1}\right) - \dfrac{\pi}{4}\right|, & GO_1 \le 0 \end{cases} \tag{4}$$

$$R = (GO_1)^2 + (GO_2)^2 \tag{5}$$

where GO_1, GO_2 are the increments of the two object values respectively. $\alpha_1 = 0.1$ and $\beta = 0.4$ are used in the present Note.

The corresponding fitness function is expressed as

$$FF = \exp[F(GO_1, GO_2)] \tag{6}$$

Usually there are some constraints in an optimization problem. These constraints can be satisfied by adding a penalty function to the resultant objective function F. If the constraints are $\Psi_i \ge D_i$, $i = 1, 2, \ldots, m$, the resultant objective function with constraints can be written as

$$F = F \cdot \prod_{i=1}^{m} P_i, \qquad P_i = \begin{cases} e^{A_i(D_i - \Psi_i)}, & \Psi_i < D_i \\ 1, & \Psi_i \ge D_i \end{cases} \tag{7}$$

Received 17 August 2002; revision received 21 September 2002; accepted for publication 19 January 2003.

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Numerical optimization experiences show that the optimum-searching algorithm has significant effects on results in deterministic optimization. With the fast development of computer technology, direct searching algorithms are becoming favorable to engineers. They are simpler than other ones, such as gradient and second derivative methods. Powell method[2] is chosen as an optimum-searching algorithm in present deterministic optimization calculation.

Holland suggested the genetic algorithm,[3] which is stochastic, and the global optimization algorithm based on natural selection and evolution behavior. The design variables are represented as individual members of a population. Features and parameters of the design are coded with some coding techniques and to form "chromosomes" that describe individual design variables. Individuals are evaluated for a fitness value, which is a measurement of individual quality. Selection, crossover, and mutation operators are called genetic operators in GA. The process of selection, mutation, crossover, evaluation, and reproduction are repeated until the convergence of a suitable solution to the problem is achieved.

In the present Note decimal coding and game selection strategy are used. The parameters are population size 40, generation number 40, crossover probability 0.6, and mutation probability 0.4.

The computational cost of applying GA is generally found to be much higher than that of alternative approaches. This is particularly evident if Euler or Navier–Stokes flow analysis is employed. Parallel computing is therefore often required to allow a reasonable total computing time. In the present note parallel GA computations are made within cluster personal computers under an massage passing interface (MPI) [or parallel virtual machine (PVM)] environment. A network of three PCs connected via ethernet under PVM is used in two-dimensional calculation. A network of 80 PCs connected via ethernet under MPI is used in three-dimensional calculation.

Flowfield Calculation

In flowfield calculation Euler equations are used as governing equations for both two- and three-dimensional examples. The nondimensional compressible three-dimensional Euler equations in conservation form can be written as

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial \xi}+\frac{\partial F}{\partial \eta}+\frac{\partial G}{\partial \varsigma}=0 \tag{8}$$

The finite volume method is used to discretize Eq. (8), and Van Leer's scheme is used to discretize the inviscid flux vector. The MUSCL method is used to obtain a second-order accuracy. For time integration of Eq. (8), a lower-upper factorization method is used. In the following calculations drag C_D is equal to wave drag C_{D_W} because Euler equations are taken as governing equations.

Electromagnetic-Field Calculation

A two-dimensional bidisciplinary (aerodynamics/electromagnetics) optimization is made. The evaluation of RCS is based on the numerical solution of time-domain two-dimensional Maxwell equations. The TM mode of two-dimensional Maxwell equations can be written as[4]

$$Q_t + F_\xi + G_\eta = 0 \tag{9}$$

where

$$Q=\frac{1}{J}\cdot\begin{bmatrix}\varepsilon_0 E^z\\ \mu_0 H^x\\ \mu_0 H^y\end{bmatrix},\qquad F=\frac{1}{J}\cdot\begin{bmatrix}\xi_y H^x-\xi_x H^y\\ \xi_y E^z\\ -\xi_x E^z\end{bmatrix}$$

$$G=\frac{1}{J}\cdot\begin{bmatrix}\eta_y H^x-\eta_x H^y\\ \eta_y E^z\\ -\eta_x E^z\end{bmatrix} \tag{10}$$

$J=\xi_x\eta_y-\xi_y\eta_x$, ε_0, and μ_0 are the permittivity and permeablility coefficients respectively in free space.

The finite volume method and Steger–Warming's flux-splitting scheme are used to discretize Eq. (9). More detailed information for solving the boundary-value problem of Eq. (9) can be found in Ref. 4. By using the equivalence principle,[5] the obtained near-field scattered solution is converted to the far-field solution, which is transformed to frequency domain using fast Fourier transformation. The RCS, which is usually defined in frequency domain, can then be calculated.[4]

Grid Generation and Representation of an Airfoil Shape

In the optimization process the airfoil shape varies continuously, and a lot of iteration of field solution is required. Thus, a fast, robust, and high qualitative grid-generation method is essential. A transfinite interpolation grid-generation method applying B-spline curve lines and surfaces is used. The basic considerations are as follows:

1) Original B-spline method can produce a high qualitative curve surface grid, but the grid boundaries are only similar to and not coincident with the initial boundaries. Applying transfinite interpolation method, they can be completely coincident.

2) Transfinite interpolation method can generate the smoothest grid with continuous second derivatives when B spline is used as blending function, especially when the inverse algorithm of B-spline curve surface generation is used, in which the boundary conditions of tangential vector at end points of boundary and torque vector at four corners are used. These vectors can be controlled and adjusted to meet the requirement to the grid.

In three-dimensional calculation a two-dimensional grid is generated in a streamwise direction by using this method, and the spanwise grid topogy is of H type.

The coordinates of an airfoil contour are taken as the design variables, and the analytic shape function[6] is used to vary the shape of the airfoil,

$$\bar{y}=\bar{y}_b(\bar{x})+\sum_{i=1}^{n}a_i s_i(\bar{x}) \tag{11}$$

where $\bar{y}_b\,(\bar{x})$ is the baseline profile; $\bar{x}$, $\bar{y}$ are nondimensional coordinates; $s_i\,(\bar{x})$ is one of the shape functions to perturb systematically the baseline profile; and undetermined coefficients a_i are taken as the design variables.

Numerical Results and Discussions

Two-Dimensional Case Examples

Case 1

Airfoil NACA0012 is chosen as a baseline airfoil. It is desired to modify its shape to minimize its drag C_D and at the same time to maximize its lift C_L under the condition of $M_\infty=0.75$, $\alpha=2.57$ deg and the constraint of $(t/C)_{\max}\geq(t/C)_{\max 0}$. For comparison, this example is also calculated using the LWO method with two different combinations of weighted coefficients. Both OFCM and LWO calculations are carried out using both deterministic (Powell) method and genetic algorithm. Table 1 gives the calculated results.

It can be seen that in LWO solutions if one relative increment is positive then the other is negative, and this is not our goal. The present (OFCM) method gives better compromised solutions, which meet the requirements.

Case 2

Airfoil NACA65006 at $M_\infty=1.5$, $\alpha=0$ deg is optimized to realize low observability at the leading edge and a smaller C_D at the same time under the constraints of keeping the airfoil symmetric and the maximum thickness larger than or equal to the original, that is, $(t/C)_{\max}\geq(t/C)_{\max 0}$. The calculated results are given in Table 2. The result using LWO with (0.5,0.5) is also given here for comparison.

Table 2 shows that pareto solutions are obtained in all calculations in bidisciplinary optimization and a better compromised solution is obtained using OFCM.

Three-Dimensional Case Example

Three-dimensional Euler equations are used as governing equations for flowfield calculation, and the parallel genetic algorithm with OFCM is used as the optimization method.

Table 1 Comparison of calculated results of NACA0012

NACA0012 $M_\infty = 0.75, \alpha = 2.57$ deg	C_L	Increment, %	$1/C_D$	Increment, %
Initial	0.6538	——	27.63 (1/0.03618)	——
Optimized				
LWO				
(0.5, 0.5)				
Powell	0.2887	−55.8	162 (1/0.00618)	486
GA	0.3604	−45	132.1 (1/0.00757)	378.1
(0.9, 0.1)				
Powell	1.3111	100.5	4.27 (1/0.2342)	−84.5
GA	1.1878	81.6	11.15 (1/0.089667)	−59.6
OFCM				
Powell	0.985	50.7	42.37 (1/0.0236)	53.3
GA	0.8979	37.3	48.3 (1/0.0207)	75

Table 2 Comparison of calculated results of NACA65006

NACA65006 $M_\infty = 1.5, \alpha = 0$ deg	$1/C_D$	Increment, %	$-RCS_{\theta = 180\,\mathrm{deg}}$	Increment, %
Initial	37.34 (1/0.02678)	——	5.074	——
Optimized				
OFCM				
Powell	45.25 (1/0.02210)	21.2	13.70	170
GA	44.42 (1/0.02251)	18.9	16.22	220
LWO				
GA	41.0 (1/0.024401)	9.8	42.94	746

Table 3 Comparison of calculated results of 3D case

	$M_\infty = 0.6, \alpha = 2$ deg				$M_\infty = 1.5, \alpha = 0$ deg	
3D wing BO	C_L	C_{DW}	C_L/C_{DW}	$\Delta C_L/C_{DW}$, %	C_{DW0}	ΔC_{D0}, %
Initial	0.045801	0.005155	8.8842	0	0.01177	0
Optimized						
OFCM	0.098092	0.007176	13.6702	53.9	0.015405	30.9
LWO	0.102980	0.007437	13.8462	55.9	0.015824	34.4

A three-dimensional wing plane is taken as the baseline, which has a sweep angle of leading edge $\chi = 35$ deg, an aspect ratio $\lambda = 3.9$, and a taper ratio $\eta = 0.17$. The NACA 65006 airfoil is taken as the wing-section profile. It is required to increase C_L/C_D at $M_\infty = 0.6, \alpha = 2$ deg and at the same time to reduce C_D at $M_\infty = 1.5, \alpha = 0$ deg. In the optimization process five sections are used as control sections. In each of them four coefficients a_i of Eq. (11) are used as design variables. The total number of design variables is 20. For comparison, calculation is also made using LWO with (0.5, 0.5). Table 3 gives the calculated results.

To increase C_L/C_D in subsonic speed, the three-dimensional wing has to be cambered and twisted. However, this makes the drag increase in supersonic speed. Biobjective optimization can let this increase be as low as possible. The optimized results using both LWO and OFCM methods give the same good compromised solutions, which are shown in Table 3.

Conclusions

Biobjective and bidisciplinary optimal design of high-performance airfoil and wing applying two kinds of optimization methods [deterministic optimization (DOM) and genetic algorithm (GA)] are discussed. An objective function combination method (OFCM) of constructing a resultant objective function in DOM or a fitness function in GA for biobjective optimization is suggested. Comparing to LWO, OFCM does not require user's experience of giving a suitable weighted coefficient combination. Numerical results in both two-dimensional (airfoil) and three-dimensional (wing) cases show the following:

1) All of the optimal solutions improve the original performance obviously.

2) Using the suggested OFCM, a better than or at least the same as LWO's compromised solution can be achieved without requiring the user's experience.

3) Solutions using DOM and GA are similar and comparable because their optimizations are initiated from same baseline airfoils or wings.

4) The optimization method presented in this Note can be used to optimize different kinds of initial airfoils, wings, and flow conditions. The performance of the optimized shape is improved significantly. The method works successfully and effectively.

Acknowledgment

This work is supported by National Science Foundation of China (NSFC)(10272013). The three-dimensional cases are calculated on the computer network of Laboratory for Study of Turbulence and Computer Systems (LTCS), Beijing University, People's Republic of China.

References

[1] Stadler, W., "Multicriteria Optimization in Mechanics (A Survey)," *Applied Mechanics Reviews*, Vol. 37, No. 3, 1984, pp. 277–286.

[2] Powell, M. J. D., "An Efficient Method for Finding the Minimum of a Function of Several Variables Without Calculation Derivatives," *Computer Journal*, Vol. 7, No. 2, 1964, pp. 155–162.

[3] Holland, J. H., *Adaption in Natural and Artificial Systems*, Univ. Michigan Press, Michigan, 1975.

[4] Zhu, Z. Q., Islam, Z., Zhu, Y. K., and Li, H. M., "The Numerical Optimization Computation of Fluid/Electromagnetic Fields," *Computational Fluid Dynamics Journal*, Vol. 7, No. 2, 1998, pp. 229–244.

[5] Umashankar, K., and Taflove, A., "A Noval Method to Analyze Electromagnetic Scattering of Complex Objects," *IEEE Transactions on Elect. Comp.*, EMC-Vol. 24, No. 4, 1982, pp. 405–410.

[6]Hager, J. O., Eyi, S., and Lee, K. D., "Multi-Point Design of Transonic Airfoils," Using Optimization," AIAA Aircraft Design Systems Meeting, AIAA Paper 92-4225, Aug. 1992.

Rollup and Near-Field Behavior of a Tip Vortex

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Introduction

THE vortex system shed from an aircraft wing rolls up rapidly downstream of the aircraft to form two counter-rotating vortices. These vortices may persist for some time after their generation and can induce significant upsetting rolling or pitching motions and thus pose a hazard to other aircraft that penetrate them. Moreover, tip vortices shed from helicopter rotor blades and propellers interact with following blades causing rotor noise and vibration. Therefore, to minimize the wait time of aircraft on the ground and in the air, as well as to reduce the tip-vortex-generated noise and accompanying potential hazards, the tip-vortex-wake characteristics must be measured and predicted accurately and controlled to allow the most efficient use of airport facilities and to ensure flight safety as well.

Numerous experimental, theoretical, and computational investigations have been conducted to improve the understanding of the tip-vortex structure and its dissipation or persistence. However, unlike the usual void of experimental data, a substantial effort has been invested in developing theoretical and numerical models for the rollup process of tip vortices. Excellent reviews of the underlying theories are given by Williams[1] and Spalart.[2] The bulk of the experimental effort has been mainly directed toward finding the rate of change of the tangential velocity and the strength of trailing vortices in the intermediate- or far-field regions, while addressing the issues of vortex development, stability, and breakdown. However, the dynamics of the initial rollup of a tip vortex around the wing tip and its subsequent development in the near field behind the trailing edge have not been studied sufficiently.[3–8] The near-field behavior of a tip vortex is significant in the flow behind canard wings, helicopter blades (a major noise source), sails of submarines, and propeller blades, where control of tip-vortex cavitation to reduce the noise and wear is of extreme importance.

The objective of the present study was to characterize the formation and growth of a tip vortex along the tip of a rectangular high-lift wing and its subsequent development in the near field (up to 1.5 chords downstream of the trailing edge of the wing) by using a miniature seven-hole pressure probe and particle image velocimetry at low Reynolds numbers. Special attention was focused on the effects of airfoil incidence angle, and Reynolds number, and sweep angle on the vortex strength, size, and tangential and axial velocity distributions. It is anticipated that the present measurements will add more information to the understanding of a tip vortex and its prediction.

Received 16 October 2003; revision received 3 January 2003; accepted for publication 4 January 2003.

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Experimental Apparatus and Methods

The experiment was carried out in a low-turbulence $0.9 \times 1.2 \times 2.7$ m subsonic wind tunnel. A single-element rectangular high-lift Bombardier Research and Development wing with a chord c of 50.8 cm and a span of 50.8 cm was used to generate the tip vortices. The square-edged airfoil was mounted vertically at the center of the bottom wall of the wind-tunnel test section. The origin of the coordinates was located at the leading edge of the airfoil with x, y, and z aligned with the streamwise, transverse, and spanwise directions, respectively. A miniature seven-hole pressure probe (with an outside diameter of 2.8 mm) was used to measure the three mean velocity components at $x/c = 0.5$–2.5 with the angle of attack $\alpha = 6$ and 10 deg and also at $x/c = 1.5$ with $\alpha = 2$–18 deg. The probe was calibrated *in situ* before the installation of the model. The calibration procedure followed the procedures described by Wenger and Devenport.[9] Eight pressure transducers (seven for the probe and one for tunnel reference total) were used to maximize data rate of the probe measurements system at each measurement location. The pressure signals were sampled at 500 Hz and were recorded on a 586 personal computer through a 16-bit A/D converter board. Probe traversing was achieved through a custom-built computer-controlled traversing system. Data planes taken along the tip and in the near field of the wing model had 49×49 measuring grid points with increments of $\Delta y = \Delta z = 3.2$ mm.

Double-exposure particle image velocimetry (PIV) was also employed in a 50 cm × 50 cm × 1.8 m tow tank. Sodium chloride (Fisher S127-3) was added to the water to keep the 70-μm-diam Amberlite fluorescent particles neutrally buoyant. The particles were illuminated with a laser light sheet with a thickness of 1.5 mm and a width of 30 cm. The directional ambiguity was resolved by employing a stepper-motor-driven rotating mirror image shifting technique.[10] The desired particle illuminations and duration were obtained through an electronic shutter (Uniblitz Model LS6T2), triggered by a predetermined time delay signal, in conjunction with a time delay and pulse generator (Hewlett-Packard 8015A). The output of the electronic shutter was also used to actuate the rotating mirror system. The particle images were recorded on Kodak T-Max 3200 black and white film with standard wet processing. The negatives were scanned and digitized by using a Nikon LS-1000 35-mm film scanner and were analyzed by using the AEA Technology VisiFlow PIV postprocessing software to obtain the instantaneous two dimensional velocity and vorticity distributions. Both straight and swept ($\Lambda = 25$ deg) airfoil models, Numerical-Control-machined from aluminum, were tested in the PIV experiment at a chord Reynolds number $Re = 6.7 \times 10^3$.

Results and Discussion

Figure 1 shows the typical mean velocity vectors and vorticity contours of a tip vortex at $x/c = 0.5$–2 and $\alpha = 6$ deg for $Re = 3.25 \times 10^5$. The numerical values shown in the vorticity contours indicate the normalized vorticity levels. The presence of the multiple secondary vortices (indicated by the small patches of vorticity existing between the feeding vortex sheet and the main vortex) at midchord ($x/c = 0.5$) and its wrapping around the main vortex as it progressed down the chord, as well as the subsequent development with distance aft into a progressively stronger tip vortex, can be clearly seen from the vorticity contours. At $x/c = 1.05$, the rollup was complete and a well-defined tip vortex already existed. The results also indicate that at lower Reynolds number, $Re = 1.63 \times 10^5$ (not shown here), the flow structure around the vortex core became considerably less symmetric and that the presence of secondary vortices was much more evident. Further studies with modified tip geometry are needed to quantify the role of the secondary vortices on the tip vortex size and growth.